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Effects of Sex Preference and Mortality on Family Size

MARRIED couples prefer certain minimum family size and also want to have specified number of sons and daughters within a fixed number of trials. It is shown by Sheps (1963) that for parents who want to have α sons and β daughters, the probability that they will end up with $(\alpha + \beta + h)$ children, $h \geqslant 0$, is

$$\binom{\alpha + \beta + h - 1}{\alpha - 1} p^{\alpha} q^{\beta + h} + \binom{\alpha + \beta + h - 1}{\beta - 1} p^{\alpha + h} q^{\beta}, \qquad (1)$$

where p and q are the probabilities of the birth being male and female respectively. This expression is derived with the assumption that couples can give birth to any number of children until they get the desired number in each sex. Expressions have also been derived by Sheps (1963) when couples stop once they get the desired number in each sex or stop at k-th child regardless of their distribution by sex. Mitra (1970) has derived expressions when the strategy is to have a minimum of α sons and β daughters in more than k trials. As such, a couple stops at $(k - l)$ $\{l > 0, (\alpha + \beta) < (k - l)\}$ children if the couple achieved the desired number in each sex once the $(k - l)$-th child is born or if the sex of the $(k - l)$-th child makes it impossible to reach the goal. In these works, mortality among children born is not considered. Once mortalily is considered the preference to sex will be among the living children. That is, couples will stop further conception when they achieve the minimum number of male and female surviving babies.

Extending the works of Sheps (1963) and Mitra (1970), in the first part of the paper the probability distributions under different stopping rules have been derived when the couples decide to achieve their goal within a finite number of trials and also want to have certain minimum number of children. The second part of the paper deals with the probability expressions derived by introducing the infant and early child mortality in it. This measures the combined effect of sex preference and child mortality on completed fertility.

1. Family Size under Minimum and Maximum Limits

For couples who adopt the strategy of having a minimum of children and would like to stop as soon as the first son is born within a maximum of k trials, the probability that they will give birth to exactly r children is given by

$$\begin{aligned} P_{l,k}(r, s_1) &= 0; & r < l \\ &= 1-q^l, & r = l \\ &= q^{r-1}p; & l < r < k \\ &= q^{k-1}p + q^k; & r = k \\ &= 0, & r > k \end{aligned}$$

The average number of children that will be born in the one son case is given by

$$\begin{aligned} E_{l,k}(r, s_1) &= (1-q^l)\,l + \sum_{r=l+1}^{k} rq^{r-1}p + kq^k \\ &= l + \frac{q^l}{p}(l - q^{k-l}) \qquad (2) \end{aligned}$$

which compares with $1/p$ when $l = 0$ and $k = \infty$.

In the case of two sons

$$E_{l,k}(r, s_2) = l + q^{l-1}(l - kq^{k-l}) + \frac{2q^l}{p}(1 - q^{k-l}). \qquad (3)$$

In the case of one son and one daughter

$$E_{l,k}(r, s_1, d_1) = l + \frac{q^l}{p}(1 - q^{k-l}) + \frac{p^l}{q}(1 - p^{k-l}). \qquad (4)$$

In the case of three sons

$$\begin{aligned} E_{l,k}(r, s_3) = l &+ \frac{3q^l}{p}(1-q^{k-l}) + 2q^{l-1}(l - kq^{k-l}) \\ &+ \frac{q^{l-2}p}{2}\{l(l-1) - k(k-1)q^{k-l}\}. \qquad (5) \end{aligned}$$

In the case of two sons and one daughter

$$E_{l,k}(r, s_2, d_1) = l + \frac{p^l}{q}(1-p^{k-l}) + q^{l-1}(l - kq^{k-l}) + \frac{2q^l}{p}(1-q^{k-l}) \quad (6)$$

Under the condition that couples want a specified minimum number of children and want to have some desired number in each sex when there is no maximum limit the expressions can be obtained by substituting ∞ in the place of k. The expressions thus obtained can be seen in Table 1.

1.1 PROBABILITY GENERATING FUNCTION AND MOMENT GENERATING FUNCTIONS

The probability generating function for total children $h \geqslant (\alpha + \beta)$ under no restriction for minimum and maximum children, is shown by Sheps (1963, see also Keyfitz, 1968). The probability of $(\alpha + \beta + h) = n$ children $h \geqslant 0$ is the coefficient of s^n in the generating function

$$G(s) = \left[\frac{qs}{1-ps}\right]^{\beta} + \left[\frac{ps}{1-qs}\right]^{\alpha} \quad (7)$$

where terms of lower power than $s^{\alpha+\beta}$ are disregarded. For finite l and k the generating function in the case of one can be shown as

$$G_{l,k}(s) = (l-q^l)\, s^l + \frac{pqs^{l+1}}{1-qs}\{1-(qs)^{k-l}\} + q^k s^k \quad (8)$$

Average number of children in one son case can be obtained by differentiating equation (8) with respect to s and putting $s = 1$. There does not seem to exist a general form as equation (7) under the condition of l and k for any α and β.

The variance can be obtained as below :

$$\text{var} = \left[\frac{d^2}{ds^2} G_{l,k}(s)\right]_{s=1} + \left[\frac{d}{ds} G_{l,k}(s)\right]_{s=1} - \left[\left\{\frac{d}{ds} G_{l,k}(s)\right\}_{s=1}\right]^2 \quad (9)$$

The moment generating function for the one son case with l and k as the lower and upper limits is

$$M(t) = (l-q^l)\, e^{tl} + \sum_{r=l+1}^{k} e^{tr} q^{r-1} p + q^k e^{tk}$$

$$= (1-q^l)\, e^{tl} + \frac{pq^l e^{t(l+1)}}{(1-qe^t)}\{1-(qe^t)^{k-l}\} + (qe^t)^k. \quad (10)$$

TABLE 1—AVERAGE NUMBER OF CHILDREN PER COUPLE UNDER DIFFERENT CONDITIONS

Desired number and sex of children	*no limit*	*lower limit l*	*upper limit k*	*lower limit l and upper limit k*
1	*2*	*3*	*4*	*5*
One son	$\frac{1}{p}$	$l + \left(\frac{q^l}{p}\right)$	$\frac{1}{p} - \frac{q^k}{p}$	$l + \frac{q^l}{p}(1 - q^{k-l})$
Two sons	$\frac{2}{p}$	$l + q^{l-1}\left(l + \frac{2q}{p}\right)$	$\frac{2}{p} - q^{k-1}\left(k + \frac{2q}{p}\right)$	$l + q^{l-1}(l - kq^{k-l}) + \frac{2q^l}{p}(1 - q^{k-l})$
One son and one daughter	$\frac{1}{p} + \frac{1}{q} - 1$	$l + \frac{q^l}{p} + \frac{p^l}{q}$	$\frac{1}{p} + \frac{1}{q} - 1 - \frac{p^k}{q} - \frac{q^k}{p}$	$l + \frac{q^l}{p}(1 - q^{k-l}) + \frac{p^l}{q}(1 - p^{k-l})$
Three sons	$\frac{3}{p}$	$l + \frac{3q^l}{p} + 2lq^{l-1} + \frac{l(l-1)\,q^{l-2}p}{2}$	$\frac{3}{p} - \frac{k\,(k-1)}{2}\,q^{k-2}\,p - 2kq^{k-1} - \frac{3q^k}{p}$	$l + \frac{3q^l}{p}(1 - q^{k-l}) + 2q^{l-1}(l - kq^{k-l}) + \frac{q^{l-2}p}{2}\{l(l-1) - k\,(k-1)\,q^{k-l}\}$
Two sons and one daughter	$\frac{2}{p} + \frac{p^2}{q}$	$l + \frac{p^l}{q} + q^{l-1}\left(l + \frac{2q}{p}\right)$	$\frac{2}{p} + \frac{p^2}{q} - \frac{p^k}{q} - q^{k-1}\left(k + \frac{2q}{p}\right)$	$l + \frac{p^l}{q}(1 - p^{k-l}) + q^{l-1}(l - kq^{k-l}) + \frac{2q^l}{p}(1 - q^{k-l})$

The average from the moment generating function is

$$\mu_1^1 = l + \frac{q^l}{p}(1 - q^{k-l}). \tag{11}$$

The second moment from the origin is

$$\mu_2^1 = l^2 + (2l+1)q^l - (2k+1)q^k + \frac{(2l+3)}{p} q^{l+1} - \frac{(2k+3)}{p} q^{k+1}$$

$$+ \frac{2q^{l+2}}{p^2} - \frac{2q^{k+2}}{p^2}. \tag{12}$$

The variance thus obtained is

$$\text{var} = q^l\left(2l + 1 - \frac{2l}{p}\right) - q^k\left(2k + 1 - \frac{2l}{p}\right) + \frac{(2l+3)}{p} q^{l+1}$$

$$- \frac{(2k+3)}{p} q^{k+1} + \frac{2q^{l+2}}{p^2} - \frac{2q^{k+2}}{p^2} - \frac{1}{p^2}(q^l - q^k)^2. \tag{13}$$

For values of $l = 1$ and $k = \infty$, the above expression for variance reduces to q/p^2. There does not seem to exist simple expression of *MGF* for any α and β.

1.2 AVERAGE FAMILY SIZE

The averages of family size obtainable under preferred number of children in each sex and under different conditions is given in Table 1. The second column of the table provides the expected number of children under no restriction on the minimum and maximum children. The third column gives the expected number of children under the condition of minimum children desired by the couples ($l \geqslant \alpha + \beta$). It is to be noted that for $l = \alpha + \beta$ and for $l = \alpha + \beta - 1$ the expressions under column 3 reduce to the expressions under column 2. The expected number of children under the restriction of maximum children is given under column 4 for values of $k \geqslant \alpha + \beta$. The expected number of children under the restriction of both minimum and maximum number of children is given under column 5. It is to be noted that the expressions under column 5 can be obtained by adding the corresponding expressions under columns 3 and 4 and substracting the expression under column 2. The expressions under column 5 can also be independently derived and compared. It can also be verified that the expressions under column 5 reduce to other expressions under columns 2,3 and 4 by proper substitutions for / and k .

Under the assumption that the probability of birth to be a boy is 0.5, the mean family size works out to 3 with the stopping rule of one boy and one girl when there is no lower and upper limit. For the same stopping rule, the mean number of children under different conditions is given in Table 2. From the table it can be observed that the upper limit does not influence the mean family size markedly unless it is very close to the lower limit. On the other hand, any increase in the lower limit results in a substantial increase in the mean family size.

TABLE 2—MEAN NUMBER OF CHILDREN WITH A STOPPING RULE OF ONE SON AND ONE DAUGHTER UNDER DIFFERENT LOWER AND UPPER LIMITS FOR CHILDREN

Lower limit	*Upper limit*						
1	*2*	*3*	*4*	5	*6*	*7*	∞
1	2.000	2.500	2.750	2.875	2.938	2.969	3.000
3		3.000	3.250	3.375	3.438	3.469	3.500
4			4.000	4.125	4.188	4.219	4.250
5				5.000	5.063	5.094	5.125
6					6.000	6.031	6.063
7						7.000	7.031

2. Effect of Sex Preference and Early Child Mortality on Family Size

The following are assumed in developing the probability functions representing the completed fertility and achieved living children.

(a) Gouples are fertile ;

(b) Birth control is complete ;

(c) Sex of each child is independent of the sexes of the other children born in the family ;

(d) A woman can conceive as many times as the couple desires ;

(*e*) Infant child mortality occurs in the first two years of life and later no death occurs among the children until the couples complete their fertility;

(*f*) Couples take decision in going in for the next baby when the age of the previous baby born is aged two (i.e. the survival of the baby born is established).

Assumptions (*e*) and (*f*) are not far fetched since the mortality after two years of life is comparatively low and that temporary methods of contraception are available to postpone the next pregnancy.

Let δ be the probability of death among male babies in the first two years of life and ϵ be the probability of death among female babies.

$$p_1 = p(1 - \delta)$$

$$p_2 = p\delta$$

$$q_1 = q(1 - \epsilon)$$

$$q_2 = q\epsilon.$$

For parents who want to have α living sons and β living daughters, the probability that they will end up with $(\alpha + \beta + h)$ births, $h \geqslant 0$ is

$$P_b(\alpha + \beta + h, \alpha, \beta) = \binom{\alpha + \beta + h - 1}{\alpha - 1} p_1^{\alpha} \Big[\binom{\beta + h}{\beta} q_1^{\beta} (p_2 + q_2)^h + \binom{\beta + h}{\beta + 1} q_1^{\beta+1} (p_2 + q_2)^{h-1} + \ldots + q_1^{\beta+h} \Big]$$
$$+ \binom{\alpha + \beta + h - 1}{\beta - 1} q_1^{\beta} \Big[\binom{\alpha + h}{\alpha} p_1^{\alpha} (p_2 + q_2)^h + \binom{\alpha + h}{\alpha + 1} p_1^{\alpha+1} (p_2 + q_2)^{h-1} + \ldots + p_1^{\alpha+h} \Big] \quad (14)$$

and the probability that they will end up with $(\alpha + \beta + k)$ living children $k \geqslant 0$, is

$$P_l(\alpha + \beta + k, \alpha, \beta) = \binom{\alpha + \beta + k - 1}{\beta - 1} p_1^{\alpha+k} q_1^{\beta} + \binom{\alpha + \beta + k - 1}{\alpha - 1} p_1^{\alpha} q_1^{\beta+k}$$

$$+ \binom{\alpha + \beta + k-1}{\beta-1}\binom{\alpha + \beta + k}{1}(p_2 + q_2)\, p_1^{\alpha+k}\, q_1^{\beta}$$

$$+ \binom{\alpha + \beta + k-1}{\alpha-1}\binom{\alpha + \beta + k}{1}(p_2 + q_2)\, p_1^{\alpha}\, q_1^{\beta+k}$$

$$+ \binom{\alpha + \beta + k-1}{\beta-1}\binom{\alpha + \beta + k-1}{2}(p_2 + q_2)^2\, p_1^{\alpha+k}\, q_1^{\beta}$$

$$+ \binom{\alpha + \beta + k-1}{\alpha-1}\binom{\alpha + \beta + k + 1}{2}(p_2 + q_2)^2\, p_1^{\alpha}\, q_1^{\beta+k} + \ldots$$

$$= \frac{\binom{\alpha + \beta + k-1}{\beta-1} p_1^{\alpha+k}\, q_1^{\beta}}{\{1-(p_2 + q_2)\}^{\alpha+\beta+k}} + \frac{\binom{\alpha + \beta + k-1}{\alpha-1} p_1^{\alpha}\, q_1^{\beta+k}}{\{1-(p_2 + q_2)\}^{\alpha+\beta+k}}. \quad (15)$$

It can be seen that expressions (14) and (15) reduce to expression (1) when $p_2 = q_2 = 0$ *i. e.*, $\delta = \epsilon = 0$.

2.1 PROBABILITIES UNDER DIFFERENT SET OF SEX PREFERRNCE RULES

2.1.1. The case of preference for two living sons

The probability that couples will end up with r $(= 2 + h)$ birth is

$$P_b\,(r, 2, 0) = \binom{r-1}{1}(p_2 + q_1 + q_2)^{r-2}\, p_1^2 .$$

The probability generating function is

$$PGF = \sum_{r=2}^{\infty} P_b\,(r, 2, 0)\, s^r = \frac{p_1^2\, s^2}{[1-(p_2 + q_1 + q_2)s]^2}$$

and the mean completed fertility is

$$\text{Mean} = 2/p_1$$

and

$$\text{variance} = 2\,(1 - p_1)/p_1^2$$

The probability that couples will end up with $L = (2 + k)$ living children under this stopping rule is

$$P_l\,(L, 2,\ 0, = (L-1)\, q_1^{L-2}\, p_1^2 + \binom{L}{1}(p_2 + q_2)\,(L-1)\, q_1^{L-2}\, p_1^2$$

$$+\binom{L+1}{2}(p_2+q_2)^2\,(L-1)\,q_1^{L-2}\,p_1^2+\ldots$$

$$=\frac{(L-1)\,q_1^{L-2}\,p_1^2}{\{1-(p_2+q^2)\}^L}$$

The probability generating function is

$$\sum_{L=2}^{\infty} P_l\,(L,2,0)\,s^L=\frac{p_1^2\,s^2}{\{1-q_1\,s-(p_2+q_2)\}^L}$$

The mean and variance of the achieved number of living children can be obtained from the *PGF* as

$$\text{Mean} = 2\,(p_1+q_1)/p_1$$

$$\text{Variance} = 2q_1\,(p_1+q_1)/p_1^2$$

2.1.2 The case of preference for one living son and one living daughter

The probability that couples will end up with $r\,(=1+1+h)$ births under this rule is

$$P_b(r,1,1) = \{(p_1+p_2+q_2)^{r-1}-(p_2+q_2)^{r-1}\}q_1 + \{(p_2+q_1+q_2)^{r-1}-(p_2+q_2)^{r-1}\}p_1$$

$$PGF=\frac{q_1 s}{\{1-(p_1+p_2+q_2)s\}}+\frac{p_1 s}{\{1-(p_2+q_1+q_2)s\}}-\frac{(p_1+q_1)s}{\{1-(p_2+q_2)s\}}$$

$$\text{Mean}=\frac{1}{p_1}+\frac{1}{q_1}-\frac{1}{(p_1+q_2)}$$

$$\text{Variance}=\frac{(1-p_1)}{p_1^2}+\frac{(1-q_1)}{q_1^2}-\frac{\{3-(p_1+q_1)\}}{(p_1+q_1)^2}$$

The probability that couples will have $L\,(=1+1+k)$ living children under this rule is

$$P_l\,(L,1,1) = p_1^{L-1}q_1+\binom{L}{1}(p_2+q_2)\,p_1^{L-1}q_1+\binom{L+1}{2}(p_2+q_2)^2\,p_1^{L-1}q_1$$

$$+ \dots + q_1^{L-1} p_1 + \binom{L}{1} (p_2 + q_2)\, q_1^{L-1} p_1$$

$$+ \binom{L+1}{2} (p_2 + q_2)^2\, q_1^{L-1} p_1 + \;\dots$$

$$= \frac{p_1^{L-1} q_1}{\{1-(p_2+q_2)\}^L} + \frac{q_1^{L-1}\, p_1}{\{1-(p_2+q_2)\}^L}$$

$$PGF = \frac{p_1\, q_1\, s^2}{\{1-(p_2+q_2)\}} \left[\frac{1}{\{1-(p_2+q_2)-p_1 s\}} + \frac{1}{\{1-(p_2+q_2)-q_1 s\}}\right]$$

$$\text{Mean} = (p_1/q_1) + (q_1/p_1) + 1$$

$$\text{Variance} = \frac{1}{(p_1+q_1)}\left[\frac{p_1^3}{q_1^2} + \frac{q_1^3}{p_1^2} + \frac{2p_1^2}{q_1} + \frac{2q_1^2}{p_1} - p_1 - q_1\right]$$

The expressions for the probability, moment generating function, mean and variances in the case of different other combinations of sex preference can easily be derived from (14) and (15).

2.2. MORTALITY CHANGES AND ITS EFFECTS ON COMPLETED FERTILITY AND LIVING CHILDREN

2.2.1. The case of preference for two living sons

When the probability is δ among the male births and it is the same for female births, the mean completed fertility is $MF = 2/p\ (1-\delta)$. When δ reduces to δ' the mean fertility is reduced to $MF' = 2/p\,(1-\delta')$. The proportion of reduction is

$$R = \frac{MF - MF'}{MF} = \frac{\delta - \delta'}{1 - \delta'}.$$

It can be verified that mean completed fertility is affected only by the reduction in mortality among male births and not by reduction in mortality among female births. This suggests that when there is reduction in mortality among the preferred sex of children, there will be reduction in the total number of births occurring to couples.

The mean number of living children that couples will have under this stopping rule, with δ being the mortality among the children born, is given by

$$MLC = \frac{2\{p(1-\delta) + q(1-\delta)\}}{p(1-\delta)} = 2/p.$$

Since this is free from δ, it can be understood that the mean number of living children achieved by couples will remain unaffected by any reduction in mortality among the births.

2.2.2. The case for preference for one living son and one living daughter

The mean completed fertility under this rule and when the mortality is δ among both male and female births is

$$MF = \frac{1}{p(1-\delta)} + \frac{1}{q(1-\delta)} - \frac{1}{(1-\delta)}$$

when δ is reduced to δ' the mean completed fertility is reduced to

$$MF' = \frac{1}{p(1-\delta')} + \frac{1}{q(1-\delta')} - \frac{1}{(1-\delta')}.$$

The proportionate decline is

$$R = \frac{\delta-\delta'}{1-\delta'}.$$

The mean number of achieved living children under these conditions is given by

$$MLC = \frac{p(1-\delta)}{q(1-\delta)} + \frac{q(1-\delta)}{p(1-\delta)} + 1$$

$$= \frac{p}{q} + \frac{q}{p} + 1$$

Since this is free from δ, it can be understood that mean number of living children achieved by couples remains unaffected by any simultaneous reduction in mortality from δ to δ' among male and female births.

2.3. Table 3 provides the mean and variance of the distributions of completed fertility and living children for the stopping rules (*a*) two living sons and (*b*) one living son and one living daughter. The three mortality conditions (0.15, 0.10 and 0.05) among male and female births are considered here. It is also assumed that $p = q = 0.5$. The following conclusions may be drawn from the table.

TABLE 3 — COMPLETED FERTILITV **AND** ACHIEVED NUMBER OF LIVING CHILDREN UNDER TWO PREFERENCE RULES AND DIFFERENT MORTALITY CONDITIONS

Mortality		*Preference is for two living sons*				*Preference is for one living son and one living daughter*			
Male	*Female*	*Completed fertility*		*Living children*		*Completed fertility*		*Living children*	
		Mean	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*
.15	.15	4.71	6.37	4.00	4.00	3.529	3.391	3.000	2.000
.15	.10	4.71	6.37	4.12	4.36	3.432	3.124	3.003	2.016
.15	.05	4.71	6.37	4.23	4.73	3.347	2.918	3.012	2.062
.10	.15	4.44	5.43	3.89	3.67	3.432	3.124	3.003	2.016
.10	.10	4.44	5.43	4.00	4.00	3.333	2.840	3.000	2.000
.10	.05	4.44	5.43	4.11	4.34	3.246	2.618	3.003	2.015
.05	.15	4.21	4.65	3.79	3.39	2.347	2.918	3.012	2.062
.05	.10	4.21	4.65	3.89	3.69	3.246	2.618	3.003	2.015
.05	.05	4.21	4.65	4.00	4.00	3.158	2.382	3.000	2.000

1. Disproportionate preference for sex increases the completed fertility and living children.

2. In the case of preference to one sex;

   (a) any reduction in mortality among births of the preferred sex reduces the total children born to the couples and also the achieved number of living children ;

   (b) any reduction in mortality among the births of the unpreferred sex does not affect the completed fertility of the couple but increases the achieved number of living children ; and

   (c) simultaneous reduction in mortality among male and female j births does not affect the achieved living children.

3. In case of equal preference for both sexes;

   (a) any reduction in mortality in only one sex reduces the comp-j leted fertility but increases the achieved living children;

   (b) simultaneous reduction in mortality for male and female! births reduces the completed fertility faster than the reduction) in mortality in any one sex only ; and

   (c) the simultaneous reduction in'mortality among male and femalej births does not affect the achieved living children.

Acknowledgement

The author is grateful to Dr. K. A. Pisharoti, Director, Gandhigrarnl Institute of Rural Health and Family Planning for his encouragement,! Thanks are due to Dr. K. Srinivasan, Population Officer, ECAFE,| Bangkok for suggesting the problem.

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